PRINCIPALI LIMITI NOTEVOLI

Ipotesi:

$\lim_{x\to x_0} \varepsilon(x) = 0$ e $\lim_{x\to x_0} M(x) = \pm\infty$,

con x_0 punto dell'asse reale, eventualmente $\pm\infty$.

Allora si ha:

1. $\lim_{x\to x_0} \frac{sin(\varepsilon(x))}{\varepsilon(x)} = 1$
2. $\lim_{x\to x_0} \frac{tg(\varepsilon(x))}{\varepsilon(x)} = 1$
3. $\lim_{x\to x_0} \frac{1-\cos(\varepsilon(x))}{(\varepsilon(x))^2} = \frac{1}{2}$
4. $\lim_{x\to x_0} \frac{a^{\varepsilon(x)}-1}{\varepsilon(x)} = \ln a$
5. $\lim_{x\to x_0} \frac{\lg_a(1+\varepsilon(x))}{\varepsilon(x)} = \frac{1}{\ln a}$
6. $\lim_{x\to x_0} \frac{(1+\varepsilon(x))^{\alpha}-1}{\varepsilon(x)} = \alpha$
7. $\lim_{x\to x_0} \left(1+\varepsilon(x)\right)^{1/\varepsilon(x)} = e$
8. $\lim_{x\to x_0} \left(1+\alpha\varepsilon(x)\right)^{1/\varepsilon(x)} = e^{\alpha}$
9. $\lim_{x\to x_0} \varepsilon(x) sin\frac{1}{\varepsilon(x)} = 0$
10. $\lim_{x\to x_0} \varepsilon(x)\ln\varepsilon(x) = 0$
11. $\lim_{x\to x_0} \left(1+\frac{1}{M(x)}\right)^{M(x)} = e$
12. $\lim_{x\to x_0} \left(1+\frac{\alpha}{M(x)}\right)^{M(x)} = e^{\alpha}$

Rapporto di polinomi ad infinito:
Sia

$P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$ e

$Q_m(x) = b_0x^m + b_1x^{m-1} + b_2x^{m-2} + \ldots + b_{m-1}x + b_m$.

Allora si ha:

$$\lim_{x\to+\infty}\frac{P_n(x)}{Q_m(x)}=\begin{cases} +\infty & \text{se } n>m, \dfrac{a_0}{b_0}>0 \\ -\infty & \text{se } n>m, \dfrac{a_0}{b_0}<0 \\ \dfrac{a_0}{b_0} & \text{se } n=m \\ 0 & \text{se } n<m \end{cases}$$

$$\lim_{x\to-\infty}\frac{P_n(x)}{Q_m(x)}=\begin{cases} +\infty & \text{se } n>m, \dfrac{a_0}{b_0}>0, n-m \ \text{ pari, oppure } \dfrac{a_0}{b_0}<0, n-m \ \text{ dispari} \\ -\infty & \text{se } n>m, \dfrac{a_0}{b_0}<0, n-m \ \text{ pari, oppure } \dfrac{a_0}{b_0}>0, n-m \ \text{ dispari} \\ \dfrac{a_0}{b_0} & \text{se } n=m \\ 0 & \text{se } n<m \end{cases}$$